Hysteresis modeling of the grain-oriented laminations with inclusion of crystalline and textured structure in a modified Jiles-Atherton model

Article *in* Journal of Applied Physics · January 2013

DOI: 10.1063/1.4788806

A P S Baghel
Iowa State University

AIP | Journal of Applied Physics

Hysteresis modeling of the grain-oriented laminations with inclusion of crystalline and textured structure in a modified Jiles-Atherton model

A. P. S. Baghel and S. V. Kulkarni

Citation: J. Appl. Phys. **113**, 043908 (2013); doi: 10.1063/1.4788806
View online: http://dx.doi.org/10.1063/1.4788806
View Table of Contents: http://jap.aip.org/resource/1/JAPIAU/v113/i4
Published by the AIP Publishing LLC.

JOURNAL OF APPLIED PHYSICS **113**, 043908 (2013)

Hysteresis modeling of the grain-oriented laminations with inclusion of crystalline and textured structure in a modified Jiles-Atherton model

A. P. S. Baghel and S. V. Kulkarni[a)]
Electrical Engineering Department, Indian Institute of Technology Bombay, Mumbai 400076, India

(Received 5 November 2012; accepted 4 January 2013; published online 24 January 2013)

Grain-oriented (GO) laminations owing to their crystalline and textured structure exhibit strong anisotropy in magnetic characteristics. GO laminations generally display highly steep, gooseneck, and narrow waist rolling direction (RD) hysteresis loops and complex-shaped transverse direction (TD) curves. The original Jiles-Atherton (JA) model needs improvisation while modeling such characteristics. The paper proposes a modified JA model for the hysteresis modeling of GO laminations with consideration of their crystalline and textured structure. The model is based on single crystal approximation of polycrystalline materials and modifies the anhysteretic magnetization on account of anisotropic energy. It takes into account the domain wall motion as well as domain magnetization rotation. The model provides a better prediction of RD hysteresis loops and also shows ability to characterize of TD hysteresis loops with reasonable accuracy. The model preserves simplicity of the original JA model. © *2013 American Institute of Physics*.
[http://dx.doi.org/10.1063/1.4788806]

I. INTRODUCTION

Grain-oriented (GO) materials are frequently used in electromagnetic devices due to their excellent magnetic properties in rolling direction (RD). These materials give low loss and high permeability in RD owing to their magneto-crystalline textured structure. The crystallites of Fe-Si GO laminations have their [001] easy axis close to RD and their (110) plane nearly parallel to the lamination surface; this type of crystalline structure is called Goss texture. This texture plays an important role in the magnetization process in the GO laminations.[1] Hence, these materials show highly steep, gooseneck and narrow waist hysteresis loops in RD and complex shaped curves in the transverse direction (TD).[2] Accurate representation of these types of characteristics is still a challenge to researchers and engineers. Among the existing hysteresis models in the literature, the Jiles-Atherton (JA) model and the Preisach model are most widely used.[3] The Preisach model assumes the material as an ensemble of independent particles. These particles have a simple relay-like hysteresis mechanism.[4]

Although the Preisach model is more accurate, its complex mathematical formulation is the main restriction in numerical implementations. On the other hand, the JA model is based on the magnetization through the domain wall motion with pinning effects.[5,6] Main advantages of the JA model are fewer parameters characterizing the B-H curve and its amenability with the finite element method.[7] The model basically describes the hysteresis characteristics of soft magnetic materials which are isotopic in nature. However, the model can lead to significant inaccuracies when used for hysteresis modeling of GO laminations as discussed in Refs. 8 and 9. Several efforts have been made in the literature to include crystalline and textured structure of magnetic materials in the JA model.[10–13] The JA model is extended for the inclusion of anisotropy and texture effects for the fiber-textured materials via its anhysteretic magnetization in Refs. 10–12. The extended model is particularly useful for hard magnetic materials which have larger coercive fields. On the other hand, GO laminations generally have very low coercive field and high permeability. Hence, the model does not show the ability to approximate highly steep, gooseneck and narrow waist curves of GO laminations. Another modification in the model for the crystalline and textured structured magnetic materials has been reported recently in Ref. 13. The modified model uses the Brillion function to represent the anhysteretic magnetization. It gives a better representation of the RD hysteresis loop but does not have any information about the hysteresis loops in directions other than RD. Differences in the slopes of the lower and upper branches of the loop are attributed to the asymmetric domain magnetization rotation during saturation and desaturation in Ref. 14. Asymmetry in the slopes of the branches of a measured curve of an M4 material is shown in Fig. 1. In the original model, the magnetization (reversible and irreversible) components are based on domain wall motion and it does not take into account domain magnetization rotation.

This paper proposes a modified JA model for the hysteresis modeling of GO laminations. The model is based on a single crystal approximation of polycrystalline GO laminations. Crystalline and textured nature of GO materials is included in the model by an appropriate energy function. The energy function depends on the angle between the magnetization and RD, and it shows the highest energy at 55° (approximately) from RD. The anhysteretic magnetization has been modified to take into account the additional (anisotropic) energy. Hence, the contribution of the domain magnetization rotation in the magnetization is included via the anisotropic energy function. This enables the model to approximate asymmetric slopes of the lower and upper branches of the loop. The main

a)Author to whom correspondence should be addressed. Electronic mail: svk@ee.iitb.ac.in.

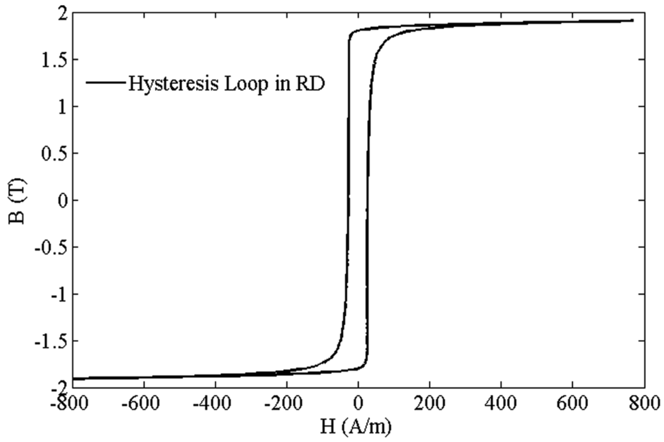

FIG. 1. Measured hysteresis loop (in RD) of a 0.27-mm-thick M4 material.

contribution of the work is that the modified model is based on the physical process of magnetization using domain wall motion and domain magnetization rotation with consideration of the crystalline and textured structure of GO laminations. The model is applied for approximation of a measured RD hysteresis loop of an M4 material. It gives reasonably accurate prediction of typical characteristics in RD. The model has also been validated on a measured TD hysteresis loop. It retains simplicity of the original model with modified anhysteretic magnetization representation.

II. HYSTERESIS MODELING FOR THE GO LAMINATIONS

A. The original JA hysteresis model

The original JA model was developed in order to describe hysteresis loops in isotropic soft magnetic materials. In the model, the anhysteretic magnetization represents the minimum energy state of the materials. It can be described in terms of three parameters a, α, and M_s.

$$\mathrm{M}_{an}(H_e) = \mathrm{M}_s\left[\coth\left(\frac{H_e}{a}\right) - \left(\frac{a}{H_e}\right)\right], \tag{1}$$

where H_e is the effective field and can be written as

$$H_e = H + \alpha M. \tag{2}$$

The JA model simulates the magnetization process through the domain wall motion with pinning effects. The hysteresis behavior is obtained by an offset from the anhysteretic magnetization using reversible and irreversible domain wall motion.[5] It is represented by the following differential equation:

$$\frac{dM}{dH} = \frac{M_{an}(H) - M_{irr}(H)}{(k\delta/\mu_0) - \alpha\Big(M_{an}(H) - M_{irr}(H)\Big)} + c\left(\frac{dM_{an}}{dH} - \frac{dM}{dH}\right), \tag{3}$$

where M is the total magnetization, H is the applied magnetic field, M_{an} is the anhysteretic magnetization, M_{irr} is irreversible magnetization, μ_0 is the magnetic permeability of free space, and δ is the directional parameter having the value $+1$ for $dH/dt > 0$ and -1 for $dH/dt < 0$. The five physical parameters of the JA model and their physical interpretations are[6]

- M_s (A/m): Saturation magnetization
- a (A/m): Form-factor or shape factor
- k (A/m): Domain wall pinning constant (irreversible magnetization component)
- c (dimensionless): Domain wall bowing parameter (reversible magnetization component)
- α (dimensionless): Mean field parameter (inter-domain coupling).

The values of these parameters need to be determined during an estimation process.[15] The first order differential equation of magnetization (1) can be integrated using a standard numerical integration technique.

B. Modified JA hysteresis model

Since the original JA model is isotropic, it cannot handle the anisotropy and textured effect of GO laminations. GO Fe-Si materials are assumed to be maintaining a cubic symmetry with axes of individual crystals along the global cubic axes.[16] However, in actual practice, they are not perfectly aligned with the global cubic axes. The magneto-crystalline anisotropy energy of a single cubic crystal can be expressed as[12]

$$E_{an} = K_0 + K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2) + K_2\alpha_1^2\alpha_2^2\alpha_3^2, \tag{4}$$

where α_1, α_2, and α_3 are the direction cosines of the magnetization vector with respect to the three crystal axes. K_0, K_1, and K_2 are anisotropy constants. The first term (K_0) represents the magnetocrystalline energy of a crystal magnetized along the easy magnetization axis and is independent of directions in the crystal.

For an ideal Goss-textured material, the (110) plane is parallel to the surface of the sheet and the [001] direction lies along RD. The magnetic field in GO laminations are generally analysed in 2-dimensions because of their small thickness. A single lamination model is shown in Fig. 2. Two dimensional description of the magnetocrystalline energy is generally used for GO laminations.

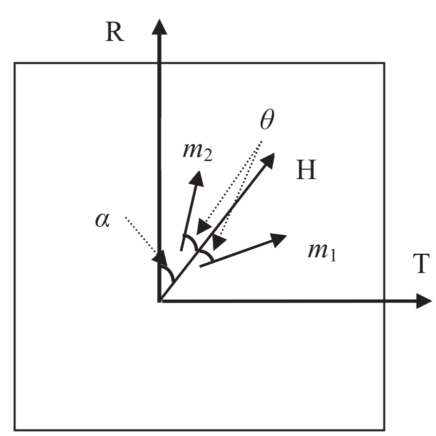

FIG. 2. A 2-D lamination model.

The direction cosines of the magnetization vector can be expressed in the polar coordinates as[17]

$$\left.\begin{aligned}\alpha_1 &= \frac{1}{\sqrt{2}}\sin\varphi\\ \alpha_2 &= \frac{1}{\sqrt{2}}\sin\varphi\\ \alpha_3 &= \cos\varphi\end{aligned}\right\}, \tag{5}$$

where φ is the angle between the magnetization direction and RD. Now, the anisotropy energy expression can be rewritten as

$$E_{an} = K_0 + K_1\left(\cos^2\varphi\cdot\sin^2\varphi + \frac{\sin^4\varphi}{4}\right) + K_2\frac{\cos^2\varphi\cdot\sin^4\varphi}{4}. \tag{6}$$

The anisotropy constants K_0 and K_2 can be neglected without affecting accuracy significantly.[17] The anisotropy energy, as a function of φ (the angle between the magnetization direction and RD), is shown in Fig. 3. The easiest direction of the magnetization is RD and the hardest direction is approximately at 55° to RD as shown in the figure.

In the presence of anisotropy, the energy equation becomes

$$E = -\mu_0 M_s\cdot(H+\alpha M) + E_{an}. \tag{7}$$

Due to symmetry in the anisotropy energy, analysis of the magnetic properties needs to be done only from 0° to 180°. Two magnetic moments m_1 and m_2, as shown in Fig. 2, are taken in the plane of the lamination. They can be at any angle (from 0° to 180°) from RD. The anisotropic energies of the two magnetic moments m_1 and m_2, can be given as

$$E_{an}(1) = K_1\left(\cos^2\varphi_1\cdot\sin^2\varphi_1 + \frac{\sin^4\varphi_1}{4}\right), \tag{8}$$

$$E_{an}(2) = K_1\left(\cos^2\varphi_2\cdot\sin^2\varphi_2 + \frac{\sin^4\varphi_2}{4}\right), \tag{9}$$

where, $\varphi_1 = \theta - \alpha$ and $\varphi_2 = \theta + \alpha$, and α is the angle between RD and the applied field direction, and θ is the angle between the applied field and the magnetic moments.

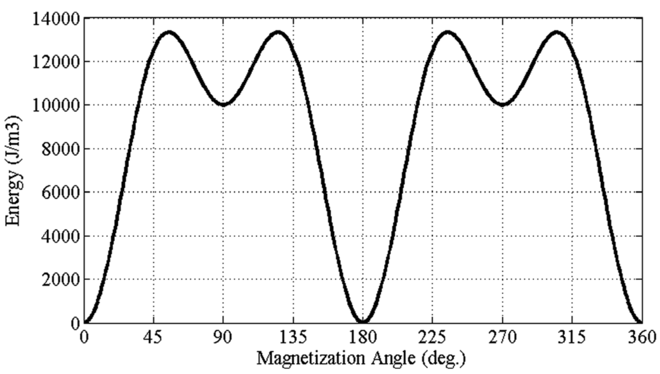

FIG. 3. Anisotropy energy Vs magnetization angle.

TABLE I. Model parameters.

Symbol	Material parameters
M_s (A/m)	1.5×10^6
a (A/m)	150
k (A/m)	200
α	1.0×10^{-3}
c	0.36
K_1 (J/m^3)	4.8×10^4

The anhysteretic magnetization can be expressed as a function of the magnetized direction as[10]

$$M_{an} = M_s\frac{\int_0^\pi \exp((E(1)+E(2))/k_BT)\sin\theta\cos\theta\, d\theta}{\int_0^\pi \exp((E(1)+E(2))/k_BT)\sin\theta\, d\theta}, \tag{10}$$

where energy terms can be expressed as given in Eq. (7)

$$E(i) = \mu_0 M_s(H+\alpha M)\cos\theta + E_{an}(i). \tag{11}$$

The anhysteretic magnetization Eq. (10) can be solved using some standard numerical integration techniques. The model now needs six parameters with an additional anisotropic parameter K_1. The hysteresis loop can be computed using the algorithm given below.

- Inputs for the model: (field data (H), time t, angle θ, and all six parameter of the model (M_s, k, a, c, α, K_1))
- Initialization $t = 1$: Δt: T (T = time period, Δt = time step)
- Input [$H(t)$, $H(t+\Delta t)$, $B(t)$]

$$\begin{aligned}\Delta H &= H(t+\Delta t) - H(t)\\ M(t) &= B(t)/\mu_0 - H(t)\\ H_e(t) &= H(t) + \alpha M(t)\end{aligned}$$

- δ is the directional parameter defined as

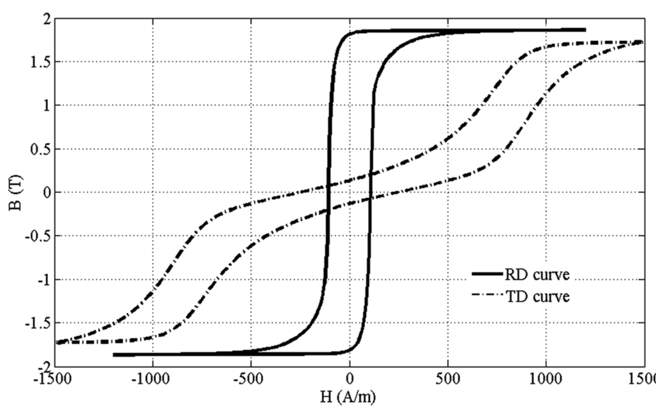

FIG. 4. Computed major hysteresis loops (for RD and TD magnetizations).

TABLE II. Optimized parameters for the original JA model.

Symbol	Optimized model parameters
M_s (A/m)	1.54×10^6
a (A/m)	17
k (A/m)	73
α	7.2×10^{-5}
c	0.36

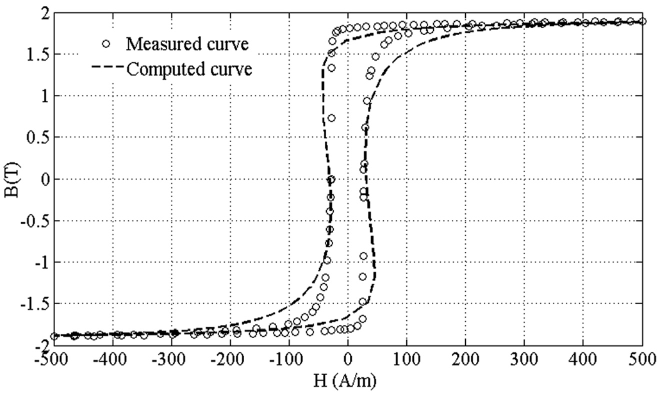

FIG. 5. Computed and measured hysteresis loops using the original JA model.

$$\left.\begin{aligned}\delta &= +1 \text{ for } \quad dH/dt > 0\\ \delta &= -1 \text{ for } \quad dH/dt < 0\end{aligned}\right\}$$

- $M_{an}(t)$ can be calculated using a standard integration technique (trapezoidal rule) in Eq. (10)
- $M(t+\Delta t)$ can be calculated using the following equation:
- $M(t+\Delta t) = M(t) + \frac{(1-c)(H(t+\Delta t)-H(t))(M_{an}(t)-M(t))}{\delta k-\alpha(M_{an}(t)-M(t))} + c(M_{an}(t+\Delta t) - M_{an}(t))$
- B field at the next time step $(t+\Delta t)$ is

$$B(t+\Delta t) = \mu_0(M(t+\Delta t) + H(t+\Delta t))$$

- If $t < \mathrm{T}$, repeat the above procedure, otherwise stop.

TABLE III. Optimized parameters for the modified JA model.

Symbol	Optimized model parameters (RD Loop)	Optimized model parameters (TD Loop)
M_s (A/m)	1.5×10^6	1.25×10^6
a (A/m)	57	600
k (A/m)	38	70
α	3.8×10^{-5}	7.1×10^{-4}
c	0.3	0.45
K_1 (J/m^3)	6.5×10^4	2.5×10^3

The modified model retains the simplicity of the original model and it can also be implemented in its inverse form in FEM analysis.

III. RESULTS AND DISCUSSIONS

The modified JA model is used for computation of the hysteresis curves in different directions. The model parameters are given in Table I.

The computed curves for RD and TD using the model are shown in Fig. 4. It can be observed from the figure that the model can also be used to predict hysteresis curves in different directions. The complex shaped TD hysteresis loop in the figure has a shape that is similar to the measured curve reported in Ref. 2.

The original JA model is applied to a measured hysteresis loop of a grain-oriented (M4) material. The measurement is carried out using a standard Epstein frame instrument (Model: BROCKHAUS MPG 100D). It consists of a bridge and a device comprising of a sine wave generator, an amplifier and signal processing units. Using the sine wave generator and the amplifier, one can choose frequency and flux density values very precisely. Lamination samples of standard dimensions are used for the measurement. The thickness of the material is 0.27 mm, the length is 305 mm, and the width is 30 mm. The measurement frequency was set to 10 Hz. The effects of classical eddy currents and anomalous losses on the hysteresis loop are neglected. The model parameters for the measured

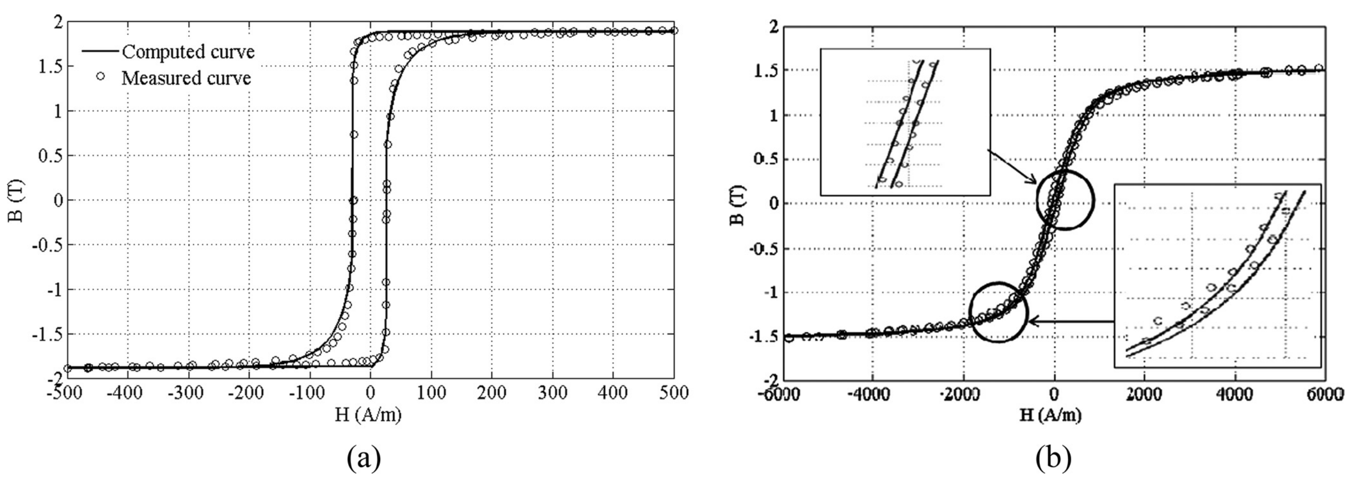

FIG. 6. Measured and computed hysteresis loops (a) RD and (b) TD.

curve have been obtained using a hybrid technique proposed in Ref. 12. The optimization procedure is repeated 15 times to ensure best fit values, and the optimized parameters are given in Table II.

The computed and measured curves are shown in Fig. 5. Errors are estimated at five reference points of the hysteresis curve as reported in Ref. 15. The maximum error between the measured and computed values is approximately 22% at points near the knee of the curve.

Higher errors in the figure indicate the inability of the original JA model to model irregular curves with narrow waists and goosenecks shapes. The results also show negative slopes at some parts of the loop which can lead to a non-physical situation as discussed in Ref. 9.

Now, the modified JA model is applied to the same RD curve and to a measured TD hysteresis loop. The optimized parameters for RD and TD hysteresis loops are given in Table III.

The computed and measured curves for RD and TD are in close agreement, as evident in Figs. 6(a) and 6(b), respectively. The error between the measured and computed curves at any point is not more than 6% for the RD curve and not more than 13% for the TD curve. Thus, the model gives a better prediction of RD hysteresis loop and also gives reasonable accuracy in case of TD curves of GO laminations.

IV. CONCLUSIONS

A modified JA model is presented to model hysteresis of GO laminations. It has been shown to give a better prediction of highly steep, gooseneck and low waist hysteresis loops in RD than the original JA model. The model includes effects of the crystalline and textured structure of the grain-oriented materials. It is based on a single crystal approximation of polycrystalline materials and it modifies the anhysteretic magnetization on account of anisotropic energy. The energy is included in the model by its 2-D approximation. The energy function depends on the angle between the magnetization and RD, and it shows the highest energy at 55° (approximately) from RD. The anhysteretic magnetization model considers the anisotropic energy. Thus, the contribution of the domain magnetization rotation in the magnetization is included via anisotropic energy function. The main contribution of the work is that the modified model is based on the physical process of magnetization via its domain wall motion and domain magnetization rotation with the consideration of crystalline and textured structure of GO laminations. The model is verified using a measured curve of an M4 material. It provides better approximation than the original JA model for RD hysteresis loops. It also shows ability to predict hysteresis loops other than RD; it has been validated on a TD hysteresis loop. This feature will be particularly useful for analysing core joints in transformers, wherein the magnetic induction B does not remain along RD. Representation of the dynamic hysteresis with consideration of dynamic losses may possibly be included in the future work.

ACKNOWLEDGMENTS

The authors would like to thank Dr. S. E. Zirka, Dnepropetrovsk National University, Dnepropetrovsk, Ukraine, for his valuable discussions and suggestions. The authors would also like to thank Nexus Electro Steel Limited and Crompton Greaves Limited for providing measured hysteresis curves in RD and TD, respectively.

[1]V. Paltanea and G. Paltanea, Mater. Sci. Forum **670**, 66 (2011).
[2]F. Fiorillo, L. R. Dupre, C. Appino, and A. M. Rietto, IEEE Trans. Magn. **38**, 1467 (2002).
[3]A. Benabau, S. Clenet, and F. Piriou, J. Magn. Magn. Mater. **261**, 139 (2003).
[4]I. D. Mayergoyz, *Mathematical Models of Hysteresis* (Springer-Verlag, New York, 1991).
[5]D. C. Jiles and D. L. Atherton, J. Magn. Magn. Mater. **61**, 48 (1986).
[6]D. C. Jiles, J. B. Thoelke, and M. K. Devine, IEEE Trans. Magn. **28**, 27 (1992).
[7]N. Sadowski, N. J. Betistela, J. P. A. Bastos, and L. M. Mazenc, IEEE Trans. Magn. **38**, 797 (2002).
[8]S. E. Zirka, Y. I. Moroz, A. J. Moses, and C. M. Arturi, IEEE Trans. Power Deliv. **26**, 2352 (2011).
[9]S. E. Zirka, Y. I. Moroz, R. G. Harrison, and K. Chwastek, J. Appl. Phys. **112**, 043916-1 (2012).
[10]A. Ramesh, D. C. Jiles, and J. M. Roderick, IEEE Trans. Magn. **32**, 4234 (1996).
[11]A. Ramesh, D. C. Jiles, and Y. Bi, J. Appl. Phys. **81**, 5585 (1997).
[12]D. C. Jiles, A. Ramesh, Y. Shi, and X. Fang, IEEE Trans. Magn. **33**, 3961 (1997).
[13]K. Chwastek and J. Szczygłowski, Arch. Electr. Eng. **60**, 49 (2011).
[14]D. Ribbenfjard and G. Engdhal, IEEE Trans. Magn. **43**, 1385 (2007).
[15]A. P. S. Baghel and S. V. Kulkarni, IET-Electr. Power Appl. **6**, 689 (2012).
[16]M. Celasko and P. Mazzetti, IEEE Trans. Magn. **5**, 372 (1969).
[17]G. H. Sirkoohi and M. A. M. Aricat, IEEE Trans. Magn. **30**, 928 (1994).